

SWE 623 Midterm Examination

Fall 2000

Name:

Signature:

October 19, 2000

This is an open book, open notes, individual examination. If you use some proof rule, it must be stated, as discussed in class. Total number of points possible is 100. Partial credit is possible, as long as they contribute towards a complete solution. The time limit is two and a half hours. This exam contains four questions on 6 pages. Page 6 has been left blank intentionally.

Question	Points
1	
2	
3	
4	
Total	

1 Write the Following Statements Using Predicate Logic (20 Points)

For the first two questions use the predicate $father(x,y)$ to mean x is the father of y .

1.1 Every grandfather is a father (5 Points)

1.2 If every person has a father, then there is a person who is the father of himself (5 Points)

1.3 N is a prime number

1.4 More than one odd integer N exists satisfying the property that N and $(N+2)$ are both prime (5 Points)

2 Prove the following Statements using Prawitz' Style Natural Deduction (30 Points)

2.1 $A \rightarrow (B \rightarrow A)$ (15 Points)

2.2 $\exists x \forall y A(x, y) \rightarrow \forall y \exists x A(x, y)$ (15 Points)

3 Precondition Postcondition Analysis for Conditionals (20 Points)

Consider the following simple code (say S):

```
x:= y;  
If( x < 0 )  
  { x := -x; }  
else  
  { Skip }
```

3.1 State the postcondition that this code accurately computes the absolute value of Y.(5 Points)

3.2 Suppose the postcondition you stated is R, then use the proof rules for conditionals to compute $Wp(S,R)$. (15 Points)

4 Precondition Postcondition Analysis for Loops (30 Points)

Given positive integers x and y , the following code (say S) computes the integer division of y/x .

```
while((n + 1)x ≤ y)
  (n := n + 1)
```

Following are given to you:

- The Pre condition Q is $(n = 0) \wedge (x > 0) \wedge (y > 0)$.
- The Post condition R is $(nx \leq y) \wedge ((n + 1)x > y)$.
- The Loop invariant P is $(nx \leq y)$
- The bound function t is $\max\{0, (y - nx)\}$.

4.1 State the five conditions necessary to show that $\{Q\}S\{R\}$ using the loop invariant P and bound function t .(5 Points)

4.2 Prove $\{Q\}S\{R\}$, using what you stated (25 Points)

(Extra Space for Question 4)